

Written Exam Economics summer 2021

**Labour Economics**

June 9, 2021 from 10h to 22h

This exam question consists of 5 pages in total (from page 2 to page 6).

Answers only in English.

**A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes**

***The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.***

**Be careful not to cheat at exams!**

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

**Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.**

## Problem 1:

Consider a model with one sector, and one type of workers. Individuals can be unemployed or employed. The labor force is constant and normalized to 1. We assume there are  $u$  unemployed workers and  $n$  employed workers, such that  $n + u = 1$ . Unemployed workers search for jobs and firms open vacancies to hire them.

We assume that each unemployed worker exerts a search effort  $e$ . We denote total job search effort exercised by the unemployed as  $u \cdot e$  and total opened vacancies as  $v$ . The number of matches resulting from the aggregated search effort and available vacancies is given by the matching function  $M(u \cdot e, v)$ , which is increasing and concave in both its arguments and homogenous of degree one. The tightness of the labor market is defined as  $\theta = \frac{v}{u \cdot e}$ . The probability that a vacancy is filled is  $q(\theta) = \frac{M(u \cdot e, v)}{v}$ . The probability that an unemployed worker exercising one unit of search effort finds a job is  $\frac{M(u \cdot e, v)}{u \cdot e} = \theta q(\theta) = f(\theta)$ .

Unlike in the Diamond-Mortensen-Pissarides (DMP) model with wage bargaining seen in class, we assume that firms pay all workers a fixed wage  $w$  (which is above the flow value of unemployment).

**Consider firms' decision:** All firms are identical, so we can consider for simplicity that there is one representative firm, which generates a product  $y$  with its workers  $n = 1 - u$ . The firm production function is hence equivalent to the aggregate production function, and is represented by a Cobb-Douglas function:  $y(n) = a \cdot n^\alpha$ , with  $\alpha \in ]0, 1[$  and  $a > 0$ . Note that this production function implies that there are decreasing returns to labor, unlike in the DMP model where production is assumed proportional to the number of workers. To hire a new worker, the firm posts a vacancy at a cost  $c$ . Jobs are exogenously destroyed at a rate  $s$ , and  $r$  is the interest rate.

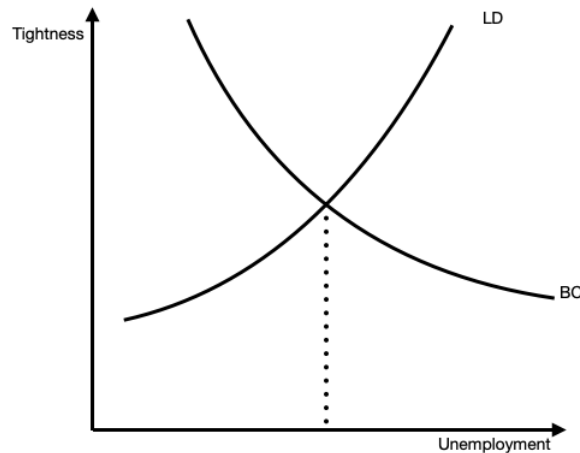
- (i). Write the Bellman equations for the value of having a vacancy  $J_v$  for the firm, and for the value of having a filled job  $J_e$  for the firm (in continuous time). Using the free-entry condition,  $J_v = 0$ , derive formally the equation for firms' labor demand. Provide an interpretation of this equation.

**Counseling program** A large intensive counseling program for unemployed workers is created. When they become unemployed, a fraction  $\beta$  ( $\beta \in ]0, 1[$ ) of workers receives it, and keeps receiving it as long as they are unemployed. Let's denote  $u_1$  the number of treated unemployed workers (i.e. those receiving the counseling program), and  $u_0$  the number of non-treated unemployed workers, such that  $u = u_0 + u_1$ . We assume that all unemployed workers keep exerting the same effort  $e$ , but the program increases by  $\gamma$  ( $\gamma > 1$ ) the productivity of their search effort, such that we consider that treated workers now exert the effort  $\gamma e$ .

- (ii). Let's consider the equilibrium of the labor market, with the counseling program. Write down the expression for the time derivatives  $\dot{u}_0$  and  $\dot{u}_1$ . Then, using the fact that at steady state, the numbers of treated unemployed workers and of non-treated unemployed workers are constant, derive the expression for the Beveridge curve:

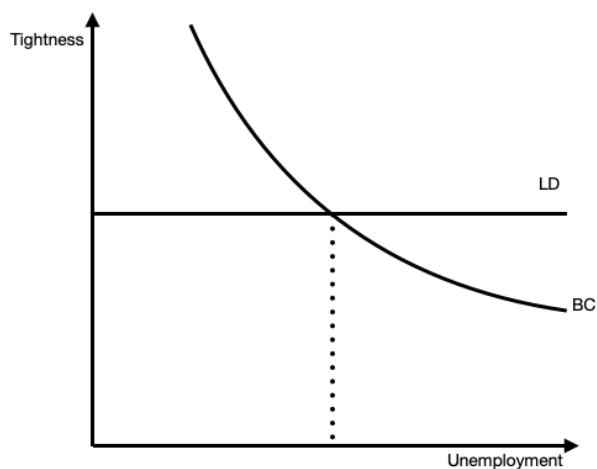
$$u = \frac{s(\beta/\gamma + 1 - \beta)}{ef(\theta) + s(\beta/\gamma + 1 - \beta)}$$

Figure 1: Labor market equilibrium, with Cobb-Douglas production function



- (iii). Consider Figure 1: it represents the Beveridge curve and of labor demand curve in the space  $(u, \theta)$ . The introduction of the counseling program corresponds to an increase in  $\gamma$ , from 1 to a level higher than 1. What happens to the Beveridge curve and to the labor demand curve when  $\gamma$  increases? How does an increase in  $\gamma$  affect the equilibrium levels of unemployment and tightness? You can use graphical representations to explain.
- (iv). For comparison, we now consider an alternative production function, where production is proportional to the number of workers, like in the DMP model:  $y(n) = a \cdot n$ , with  $a > 0$ . Figure 2 represents the corresponding labor demand curve and Beveridge curve in the space  $(u, \theta)$ . Derive the expression of firms' labor demand with this alternative production function. Then discuss what happens in that case to the equilibrium levels of unemployment and tightness, when  $\gamma$  increases. You can use graphical representations to explain.
- (v). Provide intuitions for the differences in the effect of the increase in  $\gamma$  on the equilibrium level of tightness and unemployment when the production function is Cobb-Douglas (as in question (iii)), and when it is proportional (as in question (iv)).

Figure 2: Labor market equilibrium, with proportional production function

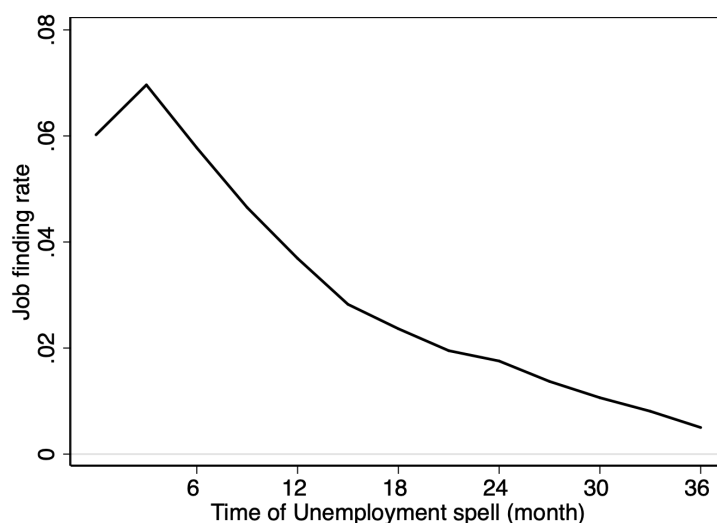


- (vi). For the rest of the problem, we always assume the production function is Cobb-Douglas, as assumed initially. Even for non-treated workers, the job finding rate is affected by the counseling program. Explain why with words.
- (vii). Assume the workers who receive the counseling program are selected randomly. Can we obtain a non biased estimate of the causal effect of the program on unemployment duration, by comparing the unemployment duration of treated workers to the unemployment duration of non-treated workers? Explain with words.

## Problem 2:

- (i). In many countries, we observe that the rate of job finding among unemployed workers decreases over time, like in Figure 3. It does not necessarily mean that the probability of finding a job for unemployed workers decreases over time. Explain with words what could be an alternative factor. Then suggest one empirical method that would help neutralize this alternative factor and estimate how the probability of finding a job for unemployed workers changes over time (several possible answers).

Figure 3: Job finding rate and time of unemployment



Notes: This figure shows the rate of unemployed workers who found a job each month, after different duration of unemployment in France.

Now, let's first consider the situation of a group of unemployed workers in a stationary environment. Assume that all individuals are identical in this group. They receive benefits  $b$  when they are unemployed and search for a job with a fixed and costless search effort (assume it is 1). Search is random: job seekers draw offers from the wage offer distribution  $H(\cdot)$ , they receive a job offer with a probability  $\lambda$ , and decide if they reject or accept the job offer. When employed, workers do not search. Jobs are exogenously destroyed at a rate  $q$ , and  $r$  is the interest rate.

- (ii). Write the Bellman equations for value of being employed, the value of being unemployed, and derive the expression of the reservation wage.
- (iii). How does the reservation wage change when  $\lambda$  increases? Show it formally, and then provide some intuition.

Now we consider that the environment of workers is non-stationary. We assume that the probability of receiving an offer is a function of the time spent unemployed  $\lambda(t)$  and drops after one year unemployed.  $\lambda(t)$  hence takes 2 values:  $\lambda_1$  before 1 year, and  $\lambda_2$  after, with  $0 < \lambda_2 < \lambda_1 < 1$ .

- (iv). Derive the expression of workers' reservation wage after one year. Then, based on the differential equation of reservation wages and on formal arguments, explain how workers' reservation wages evolve during the first year.
  
- (v). We still assume that the job arrival rate is as described in question (iv). But now, we assume that some workers never realize that their job arrival rate drops after 1 year: at all periods, they make their decision as if, for all  $t$ ,  $\lambda(t) = \lambda_1$ . Let's denote  $\alpha$  the type of workers who never realizes that the job arrival rate drops, and  $\beta$  the type of workers who knows and behaves accordingly (like described in question (iv)). Which group of workers has a higher hazard of finding a job? Which group of workers has higher expected re-employment wages? Show it formally, and then provide some intuition.